

Distributive Law:-

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

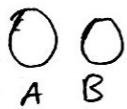
Demorgan's Law:-

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

Theorem 3

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$



$A \cap B = \phi \rightarrow$  mutually exclusive

$$P(\phi) = 0$$

Ex: Prove that  $P(A^c) = 1 - P(A)$

$$P(A \cup A^c) = 1$$

$$P(A) + P(A^c) - P(A \cap A^c) = 1$$

$\rightarrow 0$

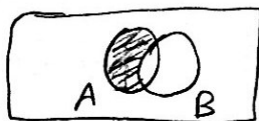
$$P(A) + P(A^c) = 1$$

$$P(A^c) = 1 - P(A)$$

① A, B Express and Venn-diagram

a) A but not B

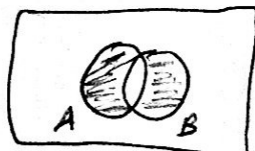
$$A - B$$



b) Either A or B but not both

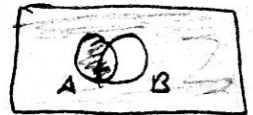
$$\rightarrow (A \cup B) - (A \cap B)$$

$$\rightarrow (A - B) \cup (B - A)$$



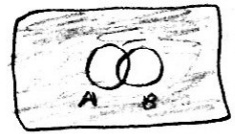
c) A or not B

$$A \cup B^c$$



d) Neither A nor B

$$(A \cup B)^c$$



② A, B, C

a) A and B but not C

$$(A \cap B) - C$$

b) only A occurs

$$\rightarrow A - (B \cup C)$$

$$\rightarrow (A - B) \cap (A - C)$$

$$\rightarrow (A - B) - C$$

③ Coin and die

a) Express

~~B~~

A: head and even

$$A = \{(H, 2), (H, 4), (H, 6)\}$$

B: prime number

$$B = \{(H, 2), (H, 3), (H, 5), (T, 2), (T, 3), (T, 5)\}$$

C: tail and odd

$$C = \{(T, 1), (T, 3), (T, 5)\}$$

b) Express

i) A or B

$$A \cup B$$

ii) B and C

$$B \cap C = \{(T, 3), (T, 5)\}$$

iii) only B

$$B - A - C = \{(H, 3), (H, 5), (T, 2)\}$$

c) Mutually Exclusive events

A, C

④  $P(H) = P(T) \times 2$

find  $P(H)$ ,  $P(T)$

$$\begin{aligned} P(H) &= \frac{2}{3} & P(H) + P(T) &= 1 \\ P(T) &= \frac{1}{3} & 2P(T) + P(T) &= 1 \\ & & 3P(T) &= 1 \\ & & P(T) &= \frac{1}{3} \\ & & P(H) &= 1 - \frac{1}{3} \\ & & P(H) &= \frac{2}{3} \end{aligned}$$

⑦ 10 boys and 20 girls  
half of boy and girls have brown eyes

$P(\text{boy or has brown eyes}) = ?$

$E_1$ : boys  $\rightarrow P(E_1) = \frac{10}{30} = \frac{1}{3}$

$E_2$ : brown eyes  $\rightarrow P(E_2) = \frac{1}{2}$

$P(E_1 \cap E_2) = \frac{5}{30} = \frac{1}{6}$

$$\begin{aligned} P(E_1 \cup E_2) &= P(E_1) + P(E_2) - P(E_1 \cap E_2) \\ &= \frac{1}{3} + \frac{1}{2} - \frac{1}{6} \\ &= \frac{2}{3} \end{aligned}$$

Theorem:- sample point =  $(c_1, c_2, \dots)$

$P(\text{sample point}) = P(c_1) \times P(c_2) \times \dots$

⑩ Find the probability of an event  
if the odds that it will occur  
are a: b that is; a to b

$P(\text{Event}) = \frac{a}{a+b}$

⑪ 3 to 2

$P(\text{Event}) = \frac{3}{5}$

⑧ die is weighted so that the  
probability of ~~number~~ a number  
appearing when the die is tossed  
is proportional to the given number

A: even no.

B: prime no.

C: odd no

a) find the probability of each sample point

b)  $P(A)$ ,  $P(B)$ ,  $P(C)$

c) ---

④

$P(1) = \frac{1}{21}$	$P(2) = \frac{2}{21}$
$P(3) = \frac{3}{21}$	$P(4) = \frac{4}{21}$
$P(5) = \frac{5}{21}$	$P(6) = \frac{6}{21}$

⑥

⑨ Prove that

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\ &\quad - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ &\quad + P(A \cap B \cap C) \end{aligned}$$

$$\begin{aligned} P(A \cup (B \cup C)) &= P(A) + P(B \cup C) - P(A \cap (B \cup C)) \\ &= P(A) + P(B) + P(C) - P(B \cap C) \\ &\quad - P((A \cap B) \cup (A \cap C)) \\ &= P(A) + P(B) + P(C) - P(B \cap C) \\ &\quad - P(A \cap B) - P(A \cap C) + \underbrace{P((A \cap B) \cap (A \cap C))}_{P(A \cap B \cap C)} \end{aligned}$$

Report  $\rightarrow 5, 6, 8, 12, 13, 15, 20$